SCIENCE

Electromagnetism & Relativity

Electric charge and Coulomb's law, electric fields and potential, circuits and Ohm's law, magnetism, electromagnetic induction, Maxwell's equations, the real crisis that led to relativity, special relativity, mass-energy equivalence, and a closing capstone on general relativity and gravity revisited a second time.

10 Chapters

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CHAPTER 1: ELECTRIC CHARGE & COULOMB'S LAW

Electromagnetism & Relativity

Course 2 · Chapter 1 · Electric Charge & Coulomb's Law

Classical Mechanics & Thermodynamics covered forces between masses. This course, the second half of the original "Classical Physics" split, opens with a different kind of force entirely — one that can both push and pull, and one whose 19th-century unification with magnetism directly set up the real crisis that led to relativity, covered later in this same course.

Electric Charge

Electric charge is a fundamental property of matter, coming in two kinds that either attract or repel. Charge is quantized in nature — every free charge is an exact whole-number multiple of the **elementary charge**, e:

```
e = 1.602176634 × 10&supminus;¹&sup9; C
```

Electric charge also obeys a real, exact conservation law: the total electric charge of an isolated system stays constant, regardless of whatever else changes within that system — charge can move from one place to another, or combine and separate in different ways, but the total amount never increases or decreases.

A Genuinely Arbitrary Naming Convention

Benjamin Franklin, working in the 1740s, coined the very term "charge" and proposed the labels "positive" and "negative" for the two kinds of electricity, based on a one-fluid theory of his own. His choice of which kind to call "positive" was, in his own era's own honest terms, an arbitrary convention — there was no physical reason at the time to prefer one label over the other.

⚠ Did Franklin "Guess Wrong"? Not Quite.

A popular version of this story claims Franklin's arbitrary guess turned out to be "backwards," since the electrons that actually carry current through a metal wire flow in the direction opposite to today's standard "conventional current" (defined as the direction positive charge would flow). This detail is real — conventional current genuinely does point opposite to real electron flow in a wire. But calling it a "wrong guess" overstates the case: a flow of positive charge in one direction has exactly the same measurable effect on a circuit as an equal flow of negative charge in the opposite direction. The convention still works correctly for every calculation; it just happens not to match which specific particle turned out to be the one actually moving, a fact nobody could have known in the 1740s, decades before the electron was even discovered.

Coulomb's Law

The force between two point charges is given by Coulomb's law:

$$F = kq_1q_2/r^2$$

Here q_1 and q_2 are the two charges, r is the distance between them, and k is Coulomb's constant, 8.9875517862×10&sup9; $N\cdot m^2\cdot C$&supminus;^{2}. Like charges repel; opposite charges attract — a genuine, structural difference from gravity, which is always attractive.

💡 A Real, Unpublished History Behind the Law

Charles-Augustin de Coulomb published this law in 1785, using a torsion balance — a bar suspended by a thin fiber, with charged metal-coated balls at each end — to measure the tiny electrostatic forces directly. But he was not the first to actually discover it: Joseph Priestley had proposed the same inverse-square relationship years earlier from his own experiments, without generalising or fully developing it, and Henry Cavendish — the same Cavendish credited in Classical Mechanics & Thermodynamics Chapter 6 with measuring the gravitational constant — had genuinely discovered the identical relationship in the early 1770s, over a decade before Coulomb, but simply never published it.

Cavendish's later, famous 1797–98 gravity experiment used the very same torsion-balance technique Coulomb would go on to use for electric charge — a real, direct instrumental link between these two inverse-square laws, discovered by overlapping figures within a few decades of each other.

Worked Example: Force Between Two Charged Spheres

Two small spheres, carrying charges of +2 μC and +3 μC, are 0.5 m apart. What force do they exert on each other?

$$F = kq_1q_2/r^2$$

$$F = (8.99\times10\text{\&sup9;}) \times (2\times10\text{\&supminus;\&sup6;}) \times (3\times10\text{\&supminus;\&sup6;}) / (0.5)^2$$

$$F \approx 0.216\ \text{N}$$

Since both charges are positive, the force is repulsive — each sphere pushes the other away.

Worked Example: Proton and Electron in a Hydrogen Atom

A hydrogen atom's electron orbits its proton at roughly the Bohr radius, $r \approx 5.29\times10\text{\&supminus;}^{11}$ m. Both particles carry a charge of magnitude e. What is the electric force between them?

$$F = ke^2/r^2$$

$$F = (8.99\times10\text{\&sup9;}) \times (1.602\times10\text{\&supminus;}^{1}\text{\&sup9;})^2 / (5.29\times10\text{\&supminus;}^{11})^2$$

$$F \approx 8.2\times10\text{\&supminus;\&sup8;}\ \text{N}$$

🔗 Why Gravity Is Negligible Inside an Atom

Using Classical Mechanics & Thermodynamics Chapter 6's own gravitational force formula ($F = Gm_1m_2/r^2$) on the same proton and electron

(m_p ≈ 1.673×10&supminus;^{2}&sup7; kg, m_e ≈ 9.109×10&supminus;^{31} kg, same distance) gives a gravitational force of only about 3.6×10&supminus;&sup4;&sup7; N — the electric force between them is roughly **2×10^{3}&sup9; times stronger** than gravity. This is a real, genuinely enormous ratio, and it's exactly why gravity is utterly negligible at atomic scales, yet completely dominates at planetary and astronomical scales: individual particles carry a fixed, sizeable charge relative to their tiny mass, while large everyday objects and planets are almost perfectly charge-neutral overall, leaving gravity as the only significant force left standing once enough matter accumulates.

Coulomb's Law and Newton's Gravity: A Direct Structural Parallel

Property	Newton's Gravity (Course 1, Ch.6)	Coulomb's Law (This Chapter)
Formula	$F = Gm_1m_2/r^2$	$F = kq_1q_2/r^2$
Distance dependence	Inverse-square	Inverse-square
Direction	Always attractive	Attractive or repulsive
Source property	Mass (always positive)	Charge (can be positive or negative)
Measured via	Torsion balance (Cavendish, 1797–98)	Torsion balance (Coulomb, 1785)

Hands-On Exercises

Exercise 1

Two point charges, +5 uC and -4 uC, are separated by 0.3 m. Calculate the magnitude of the force between them, and state whether the force is attractive or repulsive.

→ Solution

Exercise 2

Two identical point charges, each of magnitude q, are 0.2 m apart and repel each other with a force of 9 N. Using Coulomb's law, find the magnitude of q.

→ Solution

Exercise 3

Explain, in your own words, why calling Benjamin Franklin's positive/negative naming convention a "wrong guess" is a real oversimplification - what genuinely IS backwards (relative to real electron flow), and what is NOT actually a problem for how circuits are calculated using the conventional-current convention today.

→ Solution

Quick Reference

- **Elementary charge:** e = 1.602176634×10&supminus;^{1}&sup9; C; all free charge is a whole-number multiple of it
- **Conservation of charge:** total charge in an isolated system never changes
- Franklin's positive/negative labels (1740s) were a genuinely arbitrary convention, not a provably "wrong" one
- **Coulomb's law:** $F = kq_1q_2/r^2$, k ≈ 8.99×10&sup9; N·m^2·C&supminus;^{2}
- Real history: Priestley (proposed, undeveloped) → Cavendish (discovered, unpublished, early 1770s) → Coulomb (published, 1785)
- Electric force between a proton and electron is roughly 2×10^3&sup9; times stronger than gravity between them

Next chapter: Electric Fields & Potential — where the force Coulomb's law describes gets reframed as a field filling the space around a charge, and potential energy returns from Course 1's own Chapter 3 in an electrical form.

Electromagnetism & Relativity

Course 2 · Chapter 2 · Electric Fields & Potential

Chapter 1 described the force between two charges directly, the way Chapter 6 of Course 1 first described gravity between two masses. This chapter takes the same step Course 1's own gravity chapter implicitly relied on: reframing that force as a field filling the space around a charge, and reintroducing potential energy — this time in electrical form.

The Electric Field

An **electric field** describes the force a charge would experience at any point in space, per unit of its own charge:

$$E = F/q$$

For a single point charge Q, Coulomb's law from Chapter 1 gives the field's exact real form directly:

$$E = kQ/r^2$$

The electric field is measured in newtons per coulomb (N/C), equivalently volts per metre (V/m). Unlike a bare force, a field exists at every point in space regardless of whether a second charge happens to be there to feel it — it describes what *would* happen to a charge, if one were placed at that location.

💡 **Michael Faraday's Real, Untrained Genius**

The idea of a "field" filling empty space — rather than charges reaching out and acting on each other directly at a distance — comes from Michael Faraday, working in the early-to-mid 1800s. Faraday's own formal education was genuinely minimal: by his own later account, his mathematical ability "did not extend as far as trigonometry," and he was largely self-taught, having taken up reading and lectures while apprenticed as a bookbinder from age 14. Working almost entirely through visualised "lines of force" rather than equations, Faraday proposed that electric and magnetic forces genuinely extend into the empty space around a charge or magnet — an idea his own contemporaries initially rejected. James Clerk Maxwell, who later gave Faraday's field concept its full, rigorous mathematical form (Chapter 6 of this course), wrote of him with real, striking respect: that Faraday's own use of lines of force showed him to be "a mathematician of a very high order — one from whom the mathematicians of the future may derive valuable and fertile methods," despite having barely any formal mathematics at all.

Worked Example: The Field Around a Point Charge

What is the electric field 0.2 m from a +5 μC point charge?

```
E = kQ/r²
E = (8.99×10&sup9;) × (5×10&supminus;&sup6;) / (0.2)²
E ≈ 1.12×10&sup6; N/C
```

Electric Potential

Just as Course 1's own Chapter 3 defined gravitational potential energy from work done against gravity, **electric potential** (commonly called voltage) measures electrical potential energy per unit charge:

$$V = kQ/r$$

The electric potential energy of a charge q placed at a location with potential V is:

$$U = qV$$

Electric potential is measured in volts (V), where 1 V = 1 J/C — one joule of potential energy per coulomb of charge.

💡 **Alessandro Volta and the First Steady Current**

The volt is named for Alessandro Volta, who in 1800 built the voltaic pile — genuinely the first device able to produce a steady, continuous electric current, rather than the brief static discharges every earlier electrical device (like the Leyden jar) could manage. Volta's own real motivation was a scientific dispute with fellow Italian Luigi Galvani, who had proposed that a frog's leg twitching when touched by two different metals revealed a genuine "animal electricity" produced by living tissue. Volta's crucial, correct insight was that the frog's leg was acting only as a conductor and a sensitive detector — the electricity itself came from the two dissimilar metals in contact, not from anything biological. Replacing the frog's leg with brine-soaked paper between stacked pairs of different metals, Volta both refuted Galvani's specific claim and, in the same stroke, invented the world's first battery.

Worked Example: Potential and Potential Energy

Using the same +5 μC point charge from the field example, what is the electric potential at r = 0.2 m, and what potential energy would a +2 μC test charge have there?

```
V = kQ/r
V = (8.99×10&sup9;) × (5×10&supminus;&sup6;) / 0.2
V ≈ 2.25×10&sup5; V

U = qV = (2×10&supminus;&sup6;) × (2.25×10&sup5;)
U ≈ 0.45 J
```

🔗 **A Direct Real Relationship, Not a Coincidence**

Multiplying this chapter's own field result by the distance,
E × r = (1.12×10&sup6;) × 0.2 ≈ 2.25×10&sup5; V, exactly reproduces the

potential calculated above. This is the real, general relationship E = −dV/dr (the field is the negative rate of change of potential with distance), simplified for this specific case: since a point charge's own potential is exactly zero infinitely far away, the field and potential around it are directly linked by a single factor of r.

Gravity and Electricity: The Field/Potential Parallel

Property	Gravity (Course 1)	Electricity (This Chapter)
Force	$F = Gm_1m_2/r^2$	$F = kq_1q_2/r^2$
Field	$g = GM/r^2$	$E = kQ/r^2$
Potential energy	PE = mgh (near a surface)	U = qV
Sign	Always attractive	Attractive or repulsive

Hands-On Exercises

Exercise 1

Calculate the electric field 0.5 m from a +8 uC point charge.

→ Solution

Exercise 2

A +3 uC charge is placed at a point where the electric potential is 150,000 V. Calculate its electric potential energy at that location.

→ Solution

Exercise 3

Explain, in your own words, why Volta's real insight about Galvani's frog-leg experiment - that the two dissimilar metals, not the frog's own tissue, were the true source of the electricity - was the key step that led directly to inventing the first battery.

→ Solution

Quick Reference

- **Electric field:** $E = F/q = kQ/r^2$, in N/C (equivalently V/m)
- **Electric potential:** $V = kQ/r$, in volts (1 V = 1 J/C)
- **Electric potential energy:** $U = qV$
- **Relationship:** $E = -dV/dr$
- Faraday (self-taught, minimal formal mathematics) originated the field/"lines of force" concept; Maxwell later gave it rigorous mathematical form (Chapter 6)
- Volta's 1800 voltaic pile was the first steady-current source, invented while refuting Galvani's "animal electricity"

Next chapter: Circuits: Current, Resistance & Ohm's Law — where Volta's own voltaic pile finally gets to do what it was built for, driving a steady, continuous current through a real circuit.

Electromagnetism & Relativity

Course 2 · Chapter 3 · Circuits: Current, Resistance & Ohm's Law

Chapter 2 closed by naming Volta's 1800 voltaic pile as the first source of a steady, continuous current. This chapter finally puts that current to work — and along the way, meets a second real scientist whose own most important discovery was, at first, badly received.

Electric Current

Electric current is the rate at which charge flows past a point:

```
I = Q/t
```

Current is measured in amperes (A), where 1 A = 1 C/s — one coulomb of charge passing per second.

💡 **André-Marie Ampère: "The Newton of Electricity"**

The ampere is named for André-Marie Ampère, who founded the entire field of electrodynamics almost immediately after Danish physicist Hans Christian Ørsted's real 1820 discovery: a compass needle placed near a current-carrying wire deflects, meaning an electric current itself creates a magnetic field — the very connection Chapter 4 and Chapter 5 of this course build on directly (Chapter 4 covers the real, genuinely deliberate story behind Ørsted's own discovery). Ampère seized on Ørsted's discovery immediately, developing the mathematical theory (including what's now called Ampère's force law, describing the attraction or repulsion between two current-carrying wires) that turned an isolated observation into a real, predictive science. James Clerk Maxwell — already met in Chapter 2 giving Faraday's field concept mathematical form —

later called Ampère "the Newton of electricity." The ampere itself was formally adopted as the international unit of current in 1881.

Ohm's Law

For most real conductors, voltage and current are directly proportional, related by the conductor's resistance:

$$V = IR$$

Resistance (R) is measured in ohms (Ω), where 1 Ω = 1 V/A.

⚠ A Discovery That Was Coldly Received

Georg Ohm published this exact relationship in 1827, in a book titled *Die galvanische Kette, mathematisch bearbeitet* ("The Galvanic Circuit Investigated Mathematically"). It was, by real historical account, "coldly received" by his own scientific community — his colleagues at the Jesuit Gymnasium in Cologne did not value the work, and Ohm resigned his position there as a direct result. His career then genuinely stalled for years, until he found a new post at the Polytechnic School of Nuremberg in 1833 — six years after publishing the law that now carries his name. Real recognition came only much later: the Royal Society awarded him the Copley Medal in 1841 and elected him a foreign member in 1842, and he did not reach a prestigious professorship (at Munich) until 1852, just two years before his death in 1854. This joins a real, recurring pattern across this two-course project — Carnot (Classical Mechanics & Thermodynamics, Ch.8), Cavendish and Priestley (this course, Ch.1) — of major discoveries taking real, documented years or decades to be properly recognised.

Worked Example: Current Through a Resistor

A 12 V battery is connected across a 4 Ω resistor. What current flows?

```
I = V/R
I = 12/4
I = 3 A
```

Resistance of a Wire

A real conductor's resistance depends on its own material and geometry:

```
R = ρL/A
```

Here ρ (rho) is the material's resistivity (an intrinsic property, measured in Ω·m), L is the conductor's length, and A is its cross-sectional area. Copper, a real, excellent conductor widely used in wiring, has a resistivity of roughly 1.68×10&supminus;&sup8; Ω·m — compared to a genuinely good insulator like PTFE (Teflon), whose real conductivity is roughly 10^3&sup0; times lower than copper's, making it an effectively total non-conductor for any everyday electrical purpose.

Worked Example: Resistance of a Copper Wire

What is the resistance of a 2 m length of copper wire with a cross-sectional area of 1 mm^2 (1×10&supminus;&sup6; m^2)?

```
R = ρL/A
R = (1.68×10&supminus;&sup8;) × 2 /
(1×10&supminus;&sup6;)
R ≈ 0.0336 Ω
```

A real, everyday length of copper wire has genuinely tiny resistance — exactly why copper is the standard choice for household and electronic wiring, where minimising unwanted resistance (and the power it wastes as heat) matters.

Power in a Circuit

Course 1's own definition of power (P = W/t) applies directly to circuits, combined with Ohm's law into three equivalent forms:

```
P = IV = I²R = V²/R
```

Worked Example: Power Dissipated by the Resistor

Using this chapter's own 12 V, 4 Ω, 3 A example:

```
P = IV = 3 × 12 = 36 W
P = I²R = 3² × 4 = 36 W
```

Both forms agree exactly, as they must — this power is dissipated as real heat in the resistor, the same conversion of organised electrical energy into disordered thermal energy that Classical Mechanics & Thermodynamics Chapter 8's own entropy discussion covers in general terms.

Series vs. Parallel Circuits

Property	Series Circuit	Parallel Circuit
Current	Same through every component	Splits between branches
Voltage	Splits across components	Same across every branch
Total resistance	$R_{total} = R_1 + R_2 + \ldots$	$1/R_{total} = 1/R_1 + 1/R_2 + \ldots$
If one component fails (open)	Entire circuit stops	Other branches keep working

Hands-On Exercises

Exercise 1

A 9 V battery drives a current of 0.5 A through a resistor. Using Ohm's law, calculate the resistor's resistance.

→ Solution

Exercise 2

A copper wire has a resistivity of 1.68 x 10^-8 ohm.m, a length of 5 m, and a cross-sectional area of 2 x 10^-6 m^2. Calculate its resistance.

→ Solution

Exercise 3

A heating element carries a current of 5 A and has a resistance of 20 ohms. Calculate the power it dissipates, using P = I^2 R.

→ Solution

Quick Reference

- **Current:** $I = Q/t$, measured in amperes (A)
- **Ohm's law:** $V = IR$, measured in ohms (Ω)
- **Resistance of a wire:** $R = \rho L/A$
- **Power:** $P = IV = I^2R = V^2/R$
- Ørsted's 1820 discovery (current deflects a compass needle) launched Ampère's electrodynamics
- Ohm's 1827 law was coldly received; real recognition (Copley Medal) came only in 1841

Next chapter: Magnetism & Magnetic Fields — where Ørsted's own 1820 accidental discovery, already named twice in this chapter, finally gets the full treatment.

Electromagnetism & Relativity

Course 2 · Chapter 4 · Magnetism & Magnetic Fields

Chapter 3 named Ørsted's 1820 discovery twice as the trigger for everything that followed. This chapter gives it the full treatment — and, in the process, corrects a popular version of the story this course itself repeated one chapter ago.

Magnetism Before Electricity

Magnetism was studied for centuries before anyone suspected any connection to electricity. Lodestones (naturally magnetised rock) and magnetic compasses were already in real, practical use for navigation well before the science behind them was understood. William Gilbert's 1600 book *De Magnete* gave the first real, rigorous account of why a compass points north — correctly proposing that the Earth itself is a giant magnet, tested with a spherical lodestone model he called a "terrella." Before Gilbert's own real, tested explanation, some believed a compass was drawn instead to the pole star Polaris, or to a large magnetic island somewhere at the North Pole — genuine period beliefs Gilbert's own real experimental work displaced.

Ørsted's Real 1820 Discovery

Chapter 3 described Ørsted's discovery, in passing, the way it's very commonly told: an accidental observation during a lecture demonstration, a compass needle happening to twitch near a live wire. That popular version is genuinely a myth.

⚠ Correcting Last Chapter's Own Shorthand

Hans Christian Ørsted had been actively, deliberately searching for a real connection between electricity and magnetism since 1818, motivated by his own broader philosophical convictions about a fundamental unity underlying different forces of nature. His path to the discovery was genuinely difficult —

real accounts describe him as "quite confused" by his own early results during this systematic search. After roughly three further months of intensive, deliberate experimentation following his first real observation, he published his findings in 1820, demonstrating that an electric current produces a circular magnetic field around the wire carrying it. The lecture-room accident version, however dramatic, is not the real, documented history — and this course, having repeated it in Chapter 3, is correcting it directly here rather than letting it stand.

The Magnetic Field Around a Wire

A long, straight, current-carrying wire produces a magnetic field circling around it:

$$B = \mu_0 I \ / \ (2\pi r)$$

Here I is the current, r is the perpendicular distance from the wire, and μ_0 is the vacuum permeability, 4π×10&supminus;&sup7; T·m/A — conveniently, $\mu_0/(2\pi)$ simplifies to exactly 2×10&supminus;&sup7;, giving the more compact working form B = (2×10&supminus;&sup7; × I)/r. Magnetic field strength is measured in tesla (T), a unit adopted internationally in 1960 and named for Serbian-American engineer Nikola Tesla.

The field's direction follows the real right-hand rule: point your right thumb in the direction of conventional current flow, and your curled fingers show the direction the magnetic field circles around the wire.

Worked Example: Field Around a Current-Carrying Wire

What is the magnetic field 5 cm (0.05 m) from a wire carrying 10 A?

```
B = (2×10&supminus;&sup7; × I) / r
B = (2×10&supminus;&sup7; × 10) / 0.05
B ≈ 4×10&supminus;&sup5; T (40 μT)
```

For comparison, Earth's own real surface magnetic field is roughly 25–65 μT — this ordinary current-carrying wire produces a field of genuinely comparable strength, just a few centimetres away.

Force on a Moving Charge

A charge moving through a magnetic field experiences a real, distinctive force — one that acts perpendicular to *both* the charge's own velocity and the field itself, a genuine structural difference from every force this course has covered so far:

```
F = qvB
```

(for velocity perpendicular to the field), with the exact direction given by a second real right-hand rule: point fingers along v, curl toward B, and the thumb gives the force direction on a positive charge.

Worked Example: Force on a Moving Charge

A charge of +2 μC moves at 1,000 m/s perpendicular to a 0.5 T magnetic field. What force does it feel?

```
F = qvB
F = (2×10&supminus;&sup6;) × 1000 × 0.5
F = 1×10&supminus;³ N (0.001 N)
```

Force on a Current-Carrying Wire

Since a current is simply many moving charges, a current-carrying wire sitting in a magnetic field feels a real, measurable force too:

```
F = BIL
```

where L is the length of wire within the field. This is exactly the real mechanism behind Chapter 3's own Ampère's force law — each of two parallel current-carrying wires sits inside the magnetic field the *other* wire creates (using this chapter's own $B = \mu_0 I/(2\pi r)$ formula), and each therefore feels a force via F = BIL, attracting or repelling depending on whether the two currents flow in the same or opposite directions.

Worked Example: Force on a Wire

A 0.3 m length of wire carrying 5 A sits in a 0.2 T magnetic field, perpendicular to it. What force does the wire feel?

```
F = BIL
F = 0.2 × 5 × 0.3
F = 0.3 N
```

🔗 **The Real Mechanism Behind an Electric Motor**

This exact force — a current-carrying wire pushed by a magnetic field — is the real, direct physical mechanism inside every electric motor: current flows through a coil of wire sitting in a magnetic field, the resulting force (F = BIL) pushes the coil, and the coil is arranged to rotate continuously as a result. Chapter 5's own electromagnetic induction is, in a genuine sense, this exact process running in reverse.

Electric vs. Magnetic Forces: A Real Structural Difference

Property	Electric Force (Coulomb's Law, Ch.1)	Magnetic Force (This Chapter)
Acts on	Any charge, moving or stationary	Only a moving charge (or current)

Direction relative to field	Parallel or antiparallel to E	Always perpendicular to both v and B
Does work on the charge?	Yes (changes speed)	No (only changes direction)

Hands-On Exercises

Exercise 1

Calculate the magnetic field 0.1 m from a wire carrying a current of 15 A.

→ Solution

Exercise 2

A charge of +5 uC moves at 2,000 m/s perpendicular to a 0.3 T magnetic field. Calculate the force on the charge.

→ Solution

Exercise 3

Explain, in your own words, why the popular "accidental lecture demonstration" version of Oersted's discovery is a myth - what was he really doing, and for how long, before making his 1820 discovery public?

→ Solution

Quick Reference

- William Gilbert's 1600 *De Magnete*: the Earth itself is a giant magnet
- Ørsted's 1820 discovery was the result of deliberate research since 1818, not a lecture-room accident
- **Field around a wire:** $B = \mu_0 I/(2\pi r)$, measured in tesla (T)

- **Force on a moving charge:** F = qvB, perpendicular to both v and B
- **Force on a current-carrying wire:** F = BIL — the real mechanism behind Ampère's force law and every electric motor

Next chapter: Electromagnetic Induction (Faraday's Law) — where this chapter's own motor mechanism runs in reverse, and a changing magnetic field generates an electric current instead of the other way around.

Electromagnetism & Relativity

Course 2 · Chapter 5 · Electromagnetic Induction (Faraday's Law)

Chapter 4 closed with a promise: that its own motor mechanism — a current-carrying wire pushed by a magnetic field — runs in reverse. This chapter delivers on it, with Michael Faraday returning from Chapter 2's field concept to make one more real, foundational discovery.

Faraday's 1831 Discovery

On 29 August 1831, Faraday wrapped two separate coils of wire around opposite sides of an iron ring and connected one coil to a battery. He observed something genuinely subtle: a current appeared in the second, unconnected coil only at the exact moments the battery circuit was connected or disconnected — not while a steady current was already flowing. The key real insight was that a *changing* magnetic field induces a current; a constant one, however strong, does not.

Faraday's Law

Faraday's law quantifies this exactly: the induced electromotive force (EMF, effectively an induced voltage) in a circuit equals the negative rate of change of magnetic flux through it:

$$\varepsilon = -N(d\Phi/dt)$$

Here N is the number of turns in a coil, and magnetic flux Φ is:

$$\Phi = BA$$

(for a field B perpendicular to an area A). Flux measures how much magnetic field genuinely passes through a given loop — and it's a change in this quantity, however it happens (a changing field, a changing area, or a changing angle), that induces an EMF.

Lenz's Law: The Meaning of the Minus Sign

The negative sign in Faraday's law is not a mathematical afterthought — it encodes a real physical principle, known as Lenz's law: an induced current always flows in the direction that opposes the very change that produced it. If a magnet's approach is increasing the flux through a coil, the coil's own induced current creates a magnetic field opposing that increase; if the magnet withdraws, the induced current instead opposes the decrease. This is a direct, real consequence of Chapter 3's own conservation of energy (inherited from Course 1's own Chapter 3): if induction worked the other way, reinforcing rather than opposing the change, it would create energy from nothing, with no external input required.

Worked Example: EMF from a Changing Field

A 100-turn coil has an area of 0.05 m^2. A perpendicular magnetic field through it rises steadily from 0.2 T to 0.8 T over 0.5 s. What EMF is induced?

```
ΔΦ = ΔB × A = (0.8 − 0.2) × 0.05 = 0.03 Wb
ε = N(ΔΦ/Δt) = 100 × (0.03/0.5)
ε = 6 V
```

(The magnitude is 6 V; Lenz's law separately determines the induced current's own direction — opposing the rising field.)

The Faraday Disk: The First Real Generator

In that same year, 1831, Faraday built the first working homopolar generator — a conducting copper disk rotating between the poles of a magnet, with electrical contacts at the disk's centre and at its rim. As the disk spun through the magnetic field, it genuinely generated a small, continuous electric current directly from mechanical motion — the real historical beginning of every modern dynamo and generator, though Faraday's own original disk was far too inefficient to serve as any kind of practical power source.

🔗 A Motor Run Backward

Chapter 4 described an electric motor: current through a wire, sitting in a magnetic field, produces a mechanical force ($F = BIL$) that spins a coil. The Faraday disk runs the identical physical setup in reverse: mechanical motion through a magnetic field produces a current instead. Motors and generators are, at the level of the real underlying physics, the same device used in opposite directions — a genuine, elegant symmetry running straight through both this chapter and the one before it.

Joseph Henry's Real, Delayed-Credit Story

Working independently in the United States, Joseph Henry discovered both self-inductance and mutual inductance around the very same period as Faraday — some real historians credit Henry's own observations as potentially predating Faraday's, made while Henry was building electromagnets at Albany Academy. But Henry did not publish his results promptly, and Faraday's own 1831 publication came first in print. As a direct, real consequence, Faraday became — and remains — the officially recognised discoverer of electromagnetic induction, even though Henry may genuinely have observed the effect independently around the same time, or possibly even earlier.

💡 A Recurring Real Pattern in This Two-Course Project

This joins a genuine, recurring theme spanning both this course and Classical Mechanics & Thermodynamics: Cavendish's unpublished discovery of Coulomb's law (this course, Ch.1), Ohm's initially dismissed 1827 publication (Ch.3), and now Henry's own real loss of credit specifically for not publishing in time. Priority in the history of science, again and again, has genuinely depended on publication as much as on the underlying discovery itself — which is exactly why the SI unit of inductance is still named the henry (H), a real, lasting acknowledgement of Henry's own genuine, independent contribution, even without full credit for priority.

Motors and Generators: A Direct Symmetry

Property	Motor (Chapter 4)	Generator (This Chapter)
Input	Electrical current	Mechanical motion
Output	Mechanical force/motion	Electrical current
Governing formula	$F = BIL$	$\varepsilon = -N(d\Phi/dt)$
Real historical example	Any modern electric motor	The Faraday disk, 1831

Hands-On Exercises

Exercise 1

A 50-turn coil has an area of 0.02 m^2. The magnetic field through it, initially 0.6 T, drops to 0.1 T over 0.25 s. Calculate the magnitude of the induced EMF.

→ Solution

Exercise 2

A bar magnet's north pole is pushed toward a coil of wire, increasing the magnetic flux through the coil. Using Lenz's law, explain (in terms of opposing the change) what direction the induced current must flow, and why - without needing to specify a literal clockwise/counterclockwise answer.

→ Solution

Exercise 3

Explain, in your own words, why Joseph Henry lost priority credit for discovering electromagnetic induction despite possibly observing the effect independently around the same time as Faraday, or even earlier - and name at least one other real example from this two-course project where the same kind of publishing-related credit issue occurred.

→ Solution

Quick Reference

- Faraday discovered induction on 29 August 1831: only a CHANGING current/field induces a current, not a steady one
- **Faraday's law:** $\varepsilon = -N(d\Phi/dt)$, where $\Phi = BA$
- **Lenz's law:** induced current always opposes the change that produced it (a real consequence of energy conservation)
- The Faraday disk (1831) was the first real generator — a motor's own mechanism, run in reverse
- Joseph Henry independently discovered inductance around the same time but lost priority credit by not publishing first; the henry (H) unit still honours him

Next chapter: Maxwell's Equations — where Faraday's own intuitive field concept (Chapter 2), together with everything else this course has covered, gets unified into one complete mathematical theory of electricity, magnetism, and light.

Electromagnetism & Relativity

Course 2 · Chapter 6 · Maxwell's Equations: Unifying Electricity, Magnetism & Light

Every chapter so far has handed James Clerk Maxwell a piece: Faraday's field concept (Ch.2), Ørsted and Ampère's current-creates-magnetism (Ch.3–4), and Faraday's own electromagnetic induction (Ch.5). This chapter is where Maxwell assembles all of it into one complete theory — and, in doing so, makes a real, genuinely startling discovery about what light actually is.

Maxwell's Four Equations

Between 1861 and 1865, Maxwell brought together everything electricity and magnetism had revealed since Coulomb's own 1785 law into four equations, each one a real, direct descendant of a discovery already covered in this course:

Equation	In Words	Builds On
Gauss's Law for Electricity	Electric field lines begin on positive charge and end on negative charge; total flux through any closed surface is proportional to enclosed charge	Coulomb's law (Ch.1)
Gauss's Law for Magnetism	No magnetic monopoles exist — net magnetic flux through any closed surface is always exactly zero	Magnetic field lines always form closed loops (Ch.4)
Faraday's Law	A changing magnetic field induces a circulating electric field	Electromagnetic induction (Ch.5)
Ampère–Maxwell Law	Electric currents AND changing electric fields both generate magnetic fields	Ørsted & Ampère (Ch.3–4), plus Maxwell's own real addition below

Maxwell's Own Real Contribution: Displacement Current

Three of these four laws already existed, in some form, before Maxwell. His own genuine, original contribution was the fourth equation's missing half: recognising that a *changing electric field*, even with no actual current flowing, produces a magnetic field — exactly the way an actual current does. Maxwell called this term "displacement current." Adding it restored a real, elegant symmetry to the whole theory: a changing magnetic field creates an electric field (Faraday, Ch.5), and now, a changing electric field creates a magnetic field too.

A Genuinely Startling Discovery: Light Is an Electromagnetic Wave

Maxwell's four equations, combined, predict that a changing electric and magnetic field can sustain each other, propagating outward as a real, self-sustaining wave — entirely independent of the specific charges or currents that first created it. Solving his own equations for the speed of this wave gives:

$$c = 1/\sqrt{(\mu_0\varepsilon_0)}$$

Worked Example: Calculating the Speed of Light

Using μ_0 = 4π×10&supminus;&sup7; T·m/A (from Chapter 4) and ε_0 ≈ 8.854×10&supminus;^{12} F/m (the electric permittivity of free space):

```
c = 1/√((4π×10&supminus;&sup7;) ×
(8.854×10&supminus;¹²))
c = 1/√(1.1127×10&supminus;¹&sup7;)
c ≈ 3.00×10&sup8; m/s
```

This number, calculated purely from electric and magnetic laboratory constants with no reference to light or optics at all, matched the already-known, independently measured speed of light exactly.

🔗 **Maxwell's Own Real Words**

In his 1865 paper, "A Dynamical Theory of the Electromagnetic Field" — read to the Royal Society on 8 December 1864 and approved for publication on 15 June 1865 — Maxwell wrote directly of his own realisation: "*The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*" Light itself, in other words, is nothing more or less than an oscillating electric and magnetic field — the same fields this entire course has covered from Chapter 1 onward, now understood to be responsible for vision itself.

Heinrich Hertz's Real Confirmation — and His Own Real Doubt

Maxwell's prediction remained theoretical until Heinrich Hertz, between 1886 and 1889, built real apparatus to test it directly: a spark-gap transmitter (two one-metre copper wires with a small gap between them, driven by roughly 30,000 volt pulses) and a separate receiving loop that produced its own visible spark when it detected the transmitted wave. Starting in November 1887, Hertz's real, published results confirmed that these waves travelled at a finite speed exactly matching the speed of light, and that they reflected, refracted, and polarised the same way visible light does — a full experimental confirmation of Maxwell's own theoretical prediction.

⚠ **Hertz's Own Real, Ironic Underestimate**

When asked about the practical use of his own discovery, Hertz is genuinely, directly quoted as saying: "*It's of no use whatsoever ... this is just an experiment that proves Maestro Maxwell was right — we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.*"

Pressed further on possible applications, he reportedly answered simply, "Nothing, I guess." Within six years of Hertz's own real confirmation, Guglielmo Marconi had built the first practical wireless telegraphy system using exactly the waves Hertz had just called useless — the SI unit of frequency, the hertz (Hz), now honours the very discovery its own discoverer dismissed.

Hands-On Exercises

Exercise 1

Using the same formula c = 1/sqrt(mu0 epsilon0), verify the calculation yourself by computing mu0 x epsilon0 first, then taking the square root and its reciprocal, using mu0 = 4 x pi x 10^-7 and epsilon0 = 8.854 x 10^-12. Show each intermediate step.

→ Solution

Exercise 2

Explain, in your own words, what "displacement current" actually is, and why adding it to Ampere's law (Chapter 3-4) was necessary for Maxwell's four equations to have the symmetry needed to predict a genuinely self-sustaining electromagnetic wave.

→ Solution

Exercise 3

Compare Hertz's real, documented underestimate of his own discovery's usefulness to at least one other example from this two-course project of a scientist (or their contemporaries) failing to recognize the real significance of a discovery at the time it was made.

→ Solution

Quick Reference

- **Maxwell's four equations:** Gauss's law (electricity), Gauss's law (magnetism), Faraday's law, Ampère–Maxwell law
- Maxwell's own real addition: displacement current, a changing electric field also produces a magnetic field
- **Speed of electromagnetic waves:** $c = 1/\sqrt{(\mu_0\varepsilon_0)}$ ≈ 3.00×10&sup8; m/s — exactly the known speed of light
- Maxwell's 1865 paper: light itself is an electromagnetic wave, unifying electricity, magnetism, and optics
- Hertz confirmed this experimentally, 1886–89, while genuinely doubting its own practical value; Marconi commercialised it within six years

Next chapter: The Real Crisis That Led to Relativity — where Maxwell's own theory of light, which this chapter has just unified with electricity and magnetism, turns out to conflict with something else physicists believed was equally certain, setting up the real crisis Einstein would eventually resolve.

Electromagnetism & Relativity

Course 2 · Chapter 7 · The Real Crisis That Led to Relativity

Chapter 6 established that light is an electromagnetic wave. But every other wave 19th-century physicists knew of needed a medium to travel through — water waves need water, sound needs air. What, then, was light waving through? This chapter covers the real experiment that broke the answer physicists had settled on, and the real, more complicated story of what came after.

The Luminiferous Ether

Physicists proposed that space itself was filled with an invisible medium — the "luminiferous ether" — whose vibrations light waves consisted of. The real, honest problem was that the ether needed genuinely contradictory properties to work at all: it had to be thin and frictionless enough that planets and everyday objects moved through it with no detectable drag whatsoever, yet simultaneously rigid enough to support waves propagating at Maxwell's own real 3×10&sup8; m/s — a combination no known real substance could plausibly satisfy.

The Michelson-Morley Experiment (1887)

American physicists Albert A. Michelson and Edward W. Morley set out, between April and July 1887 at what is now Case Western Reserve University in Cleveland, Ohio, to directly detect Earth's own motion through this proposed ether. Their reasoning: as Earth orbits the Sun, it should move through the stationary ether, creating a real, detectable "ether wind" — light travelling with or against this wind should take a measurably different time than light travelling across it.

Their real apparatus, a Michelson interferometer, split a single light beam into two perpendicular paths, each 11 m long, then recombined them to produce an interference pattern — sensitive enough to detect the tiny predicted timing difference. To eliminate vibration, the entire apparatus floated on a pool of mercury.

⚠ A Real, Genuinely Famous Failure

Michelson and Morley expected a fringe shift of roughly 0.4 fringes as the apparatus rotated relative to the predicted ether wind — easily large enough to measure with their real instrument. What they actually measured was a shift of 0.02 fringes or less — by their own account, roughly one-fortieth of the expected displacement, and well within the range of pure measurement noise. Michelson himself reported to Lord Rayleigh that any real ether motion, if it existed at all, had to be less than one-sixth of Earth's own orbital speed. This is genuinely regarded as one of the most famous "failed" experiments in the history of physics — not because anything went wrong with the equipment, but because a confident, specific prediction came back with a real, honest, and completely unambiguous null result.

Attempted Rescues Before Einstein

The null result did not immediately overturn the ether concept — physicists first tried to save it. George FitzGerald in 1889, and Hendrik Lorentz independently in 1892, each proposed that objects moving through the ether physically contract, ever so slightly, along their own direction of motion — by precisely the amount needed to cancel out the timing difference Michelson and Morley had expected to see. At the time, this "Lorentz–FitzGerald contraction" was a genuinely ad hoc fix, invented specifically to explain away one troublesome result rather than derived from any deeper principle.

Einstein's Real Relationship to the Experiment

The popular version of this story treats Michelson and Morley's 1887 null result as the direct spark that led Einstein to invent special relativity in 1905. The real, documented history is honestly more complicated.

🔗 A Correction Worth Making Directly

Einstein's own 1905 paper cites no prior work at all — a genuinely unusual choice for a paper of its significance. When later historians investigated what had actually shaped his thinking, Einstein himself explicitly denied that the Michelson-Morley experiment was a significant direct influence on his own reasoning. What he did credit with real, considerable influence was Lorentz's own 1895 electromagnetic theory, the separate Fizeau experiment on the speed of light in moving water, and philosophical works — notably Henri Poincaré's *Science and Hypothesis*, which Einstein and his friends are documented to have closely studied and discussed over several years. The best current historical picture is that Einstein arrived at special relativity primarily through general theoretical reasoning about Maxwell's own equations (this course's own Chapter 6) and the principle of relativity itself, rather than as a direct response to any single experimental result — even one as famous as Michelson-Morley's.

Expectation vs. Reality

What Was Believed	What Was Found
Space is filled with a detectable luminiferous ether	No ether wind was detected, to a precision roughly 20× smaller than expected
Earth's motion through the ether should shift the interference pattern by ~0.4 fringes	Measured shift: 0.02 fringes or less
The M-M null result directly inspired Einstein's 1905 relativity paper	Einstein himself denied this was a significant direct influence

Hands-On Exercises

Exercise 1

Explain, in your own words, why the luminiferous ether needed to have genuinely contradictory physical properties - name both properties directly, and explain why each was necessary for the ether theory to work at all.

→ Solution

Exercise 2

Michelson and Morley expected a fringe shift of about 0.4 fringes, but measured a shift of only about 0.02 fringes. Calculate what fraction (as a ratio, e.g. "1 in X") the measured shift represents of the expected shift, and explain in one sentence why this made the null result so difficult to dismiss as ordinary measurement error.

→ Solution

Exercise 3

Explain, in your own words, why the popular story that "Michelson-Morley's null result directly inspired Einstein's theory of relativity" is a real oversimplification, and name what Einstein himself actually credited as significant influences on his own 1905 paper.

→ Solution

Quick Reference

- The luminiferous ether was proposed as light's own necessary medium, but needed impossible, contradictory properties (frictionless yet rigid)
- Michelson & Morley (1887, Cleveland): expected a ~0.4 fringe shift from Earth's motion through the ether; measured ~0.02 or less — a genuine null result
- FitzGerald (1889) and Lorentz (1892) independently proposed length contraction as an ad hoc rescue of the ether concept
- Einstein's 1905 paper cites no prior work; he himself denied Michelson-Morley was a significant direct influence, crediting Lorentz's 1895 theory,

the Fizeau experiment, and Poincaré instead

Next chapter: Special Relativity: Real Time Dilation & Length Contraction — where the Lorentz-FitzGerald contraction this chapter introduced as an ad hoc fix gets its real, derived justification, and Einstein's own actual reasoning finally takes centre stage.

Electromagnetism & Relativity

Course 2 · Chapter 8 · Special Relativity: Real Time Dilation & Length Contraction

Chapter 7 introduced Lorentz and FitzGerald's contraction as a genuine ad hoc fix, invented specifically to rescue the ether. This chapter gives it its real, principled derivation — and closes with three real, independently measured confirmations of the theory that grew from it.

Einstein's Two Postulates (1905)

Einstein's special relativity rests on two deceptively simple starting assumptions:

- **The principle of relativity:** the laws of physics are identical in every uniformly moving (non-accelerating) reference frame — no experiment performed entirely inside a smoothly moving lab can distinguish it from a stationary one.
- **The constancy of the speed of light:** light travels at c in a vacuum, regardless of the motion of whatever emitted it or whoever is measuring it.

That second postulate is a direct, real consequence of Chapter 6's own Maxwell's equations, which give a fixed speed for electromagnetic waves with no reference to any observer's own motion at all — and it's precisely this postulate that Chapter 7's own Michelson-Morley null result is genuinely consistent with, even though (as Chapter 7 established) Einstein himself denied that experiment was a significant direct trigger for his own reasoning.

The Lorentz Factor, Properly Derived

From these two postulates alone, a single factor emerges that governs every relativistic effect:

$$\gamma = 1/\sqrt{(1 - v^2/c^2)}$$

🔗 The Same Formula, a Genuinely Different Justification

This is exactly the mathematical form FitzGerald and Lorentz proposed in Chapter 7 — but there, it was an ad hoc patch invented specifically to explain away one troublesome null result. Here, the identical formula falls directly out of Einstein's own two postulates, with no reference to the ether, Michelson-Morley, or any specific experiment at all. The same equation moved, in a real, documented seventeen years, from "a convenient fix for one anomaly" to "a necessary consequence of two simple, general principles."

Time Dilation

A moving clock, observed from a stationary frame, runs slow:

$$\Delta t' = \gamma \Delta t$$

Here Δt is the "proper time" — the time interval measured by a clock at rest relative to the event itself — and Δt' is the longer time interval a stationary outside observer measures for that same interval.

Worked Example: Time Dilation at 0.8c

A spacecraft travels at 0.8c. If 10 s pass on the spacecraft's own onboard clock, how much time passes for an observer watching from Earth?

$$\gamma = 1/\sqrt{(1 - 0.8^2)}$$
$$\gamma = 1/\sqrt{(1 - 0.64)}$$
$$\gamma = 1/\sqrt{0.36} = 1/0.6 \approx 1.667$$

$$\Delta t' = 1.667 \times 10 \approx 16.7 \text{ s}$$

While 10 s pass aboard the spacecraft, roughly 16.7 s pass for the Earth-based observer — a genuinely real, measurable difference, not merely an illusion of measurement.

Length Contraction

An object moving relative to an observer is measured as shorter, along its own direction of motion, than its "proper length" (the length measured in the object's own rest frame):

$$L' = L/\gamma$$

Using the same spacecraft ($\gamma \approx 1.667$): a 100 m spacecraft, measured by the Earth-based observer while travelling at 0.8c, would measure only $L' = 100/1.667 \approx 60$ m — even though the crew aboard, in their own rest frame, still measures their own ship at the full 100 m.

Real Confirmation 1: Muon Decay

Muons, created when cosmic rays strike the upper atmosphere, decay with a real mean lifetime of just 2.2 microseconds when at rest. Classically, even travelling near light speed, the overwhelming majority should decay before ever reaching the ground — a real 1963 experiment (Frisch and Smith) calculated that only about 27 muons per hour should reach sea level in Cambridge, Massachusetts without time dilation. What they actually measured was roughly 412 muons per hour — a real, striking discrepancy resolved exactly by time dilation, with the muons' own real measured speed (roughly 0.995c) producing a measured dilation factor of 8.8±0.8, closely matching the theoretical prediction.

Real Confirmation 2: The Hafele-Keating Experiment (1971)

Joseph Hafele and Richard Keating flew four real caesium-beam atomic clocks around the world twice aboard commercial airliners — once eastward, once westward — and compared them against a stationary reference clock at the US Naval Observatory upon return.

Direction	Predicted (ns)	Measured (ns)
Eastward	−40 ± 23	−59 ± 10
Westward	+275 ± 21	+273 ± 7

Both real measured results matched the predictions from special and general relativity combined, well within experimental error. The whole experiment, genuinely, cost only about $8,000 — with $7,600 of that spent on the researchers' own plane tickets.

Real Confirmation 3: GPS Satellites

Every GPS satellite, orbiting at roughly 3,874 m/s, experiences real time dilation from its own motion, losing about 7,214 nanoseconds per day relative to a ground clock — but its greater altitude (weaker gravity, a general-relativistic effect this course doesn't derive in depth) causes it to gain roughly 45,850 nanoseconds per day, for a real net gain of about 38.6 microseconds per day.

⚠ Real, Everyday Stakes

Left uncorrected, this 38.6 μs/day drift would accumulate into a real GPS position error of roughly 11.4 km per day — making GPS navigation completely useless within hours. Engineers correct for it directly by pre-adjusting each satellite's atomic clock frequency, before launch, from the standard 10.23 MHz down to 10.22999999543 MHz — a real, tiny, deliberate offset that exactly compensates for relativity's own real effect. Every GPS-guided phone, car, or aircraft is, quite literally, running on a working engineering correction for special and general relativity, multiple times a day.

Three Real Confirmations, Compared

Confirmation	Real Measured Effect	Scale
Muon decay	412 vs. 27 predicted per hour without dilation	Particle physics
Hafele-Keating (1971)	Nanosecond-scale clock shifts, matching prediction	Commercial aircraft
GPS satellites	38.6 μs/day net drift, corrected continuously	Everyday technology

Hands-On Exercises

Exercise 1

A particle travels at 0.6c. Calculate its Lorentz factor, and then calculate how much time passes for a stationary observer if 5 s pass in the particle's own rest frame.

→ Solution

Exercise 2

A spacecraft has a proper length of 80 m and travels at 0.6c. Using the same Lorentz factor from Exercise 1, calculate its contracted length as measured by a stationary observer.

→ Solution

Exercise 3

Explain, in your own words, why the exact same Lorentz-factor formula being an "ad hoc fix" in Chapter 7 but a "necessary consequence" in this chapter is not a contradiction - what genuinely changed between the two chapters was not the formula itself, but something else. Name what that something else was.

→ Solution

Quick Reference

- Einstein's two postulates (1905): the relativity principle, and the constancy of the speed of light
- **Lorentz factor:** $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ — the same formula from Chapter 7, now derived rather than assumed
- **Time dilation:** $\Delta t' = \gamma\Delta t$ (moving clocks run slow)
- **Length contraction:** $L' = L/\gamma$ (moving objects measure shorter along their motion)
- Real confirmations: muon decay (412 vs. 27/hour), Hafele-Keating (1971, atomic clocks on airliners), and GPS (38.6 μs/day, corrected in every satellite's own clock)

Next chapter: $E=mc^2$ and Mass-Energy Equivalence — where this same 1905 theory delivers its single most famous result, and Course 1's own conservation of energy gets one final, genuinely startling extension.

Electromagnetism & Relativity

Course 2 · Chapter 9 · $E=mc^2$ and Mass-Energy Equivalence

Chapter 8 covered how motion distorts time and length. This chapter covers relativity's single most famous consequence — and, in real, documented fact, Einstein's own version of it looked a little different from the equation everyone now recognises.

Einstein's Real 1905 Paper

Barely a few months after his main relativity paper, Einstein published a short follow-up on 21 November 1905, titled "Does the Inertia of a Body Depend Upon Its Energy Content?"

⚠ Einstein Didn't Actually Write "$E=mc^2$"

Real, documented fact: Einstein's own 1905 paper never states the compact formula $E = mc^2$ at all. What it actually derives is narrower and more specific — that if a body emits energy L (as radiation), its mass decreases by L/c^2. This describes a real, physical relationship between a CHANGE in mass and a CHANGE in energy, not yet the fully general, absolute statement that all mass is a form of stored energy. The compact, modern $E = mc^2$ form, and its full generalisation, emerged only through Einstein's own later work and that of others over the following years — a real, if minor, example of a famous equation being popularly remembered in a cleaner form than its own original source actually stated.

The Full Relativistic Energy-Momentum Relation

The complete, general relationship between energy, momentum, and mass is:

$$E^2 = (pc)^2 + (mc^2)^2$$

Two familiar special cases fall directly out of this one equation:

- For an object at rest ($p = 0$): $E^2 = (mc^2)^2$, giving the famous $E = mc^2$ — an object's own "rest energy," simply from having mass at all.
- For a massless particle, like a photon ($m = 0$): $E = pc$ — light carries real energy and momentum despite having zero rest mass.

Worked Example: The Rest Energy of 1 Gram

What is the rest energy locked in just 1 g (0.001 kg) of ordinary matter?

$$E = mc^2$$
$$E = 0.001 \times (3\times10\&sup8;)^2$$
$$E \approx 9\times10^{13}\ J$$

A single gram of matter, fully converted, carries roughly 90 trillion joules — a genuinely staggering amount of energy locked inside an ordinary object's own mass, entirely independent of any chemical or nuclear reaction actually releasing it.

Real Verification: Nuclear Mass Defect

Nuclear reactions provide the real, measurable evidence for this equivalence: the combined mass of an atomic nucleus is always slightly less than the sum of its individual protons and neutrons measured separately — the missing mass, called the mass defect, is exactly the binding energy holding the nucleus together, converted via $E = mc^2$. In real nuclear fission, roughly 0.1% of uranium's own mass converts directly into usable energy.

Real Verification: The Sun

Astronomy Fundamentals (this site's own sibling course) established that the Sun fuses roughly 600 billion kg of hydrogen every second. Using this chapter's own formula in reverse — dividing the Sun's real, measured power output (3.846×10^{2}&sup6; W) by c^2 — gives:

$$m = E/c^2 = P/c^2$$

$$m = (3.846\times10^{2}\text{\&sup6;}) \;/\; (9\times10^{1}\text{\&sup6;})$$

$$m \approx 4.27\times10\text{\&sup9;}\ \text{kg/s}$$

This matches the Sun's own real, independently measured mass-energy conversion rate of roughly 4.26 billion kg/s almost exactly — only about 0.7% of the 600 billion kg of hydrogen fused per second actually converts to energy; the rest remains as the resulting helium's own mass.

Real Verification: The Trinity Test

The real 1945 Trinity nuclear test used a plutonium core of roughly 6.15 kg, of which about 1 kg genuinely underwent fission — and the real, measured mass converted into energy was almost exactly one gram.

🔗 **Checking the Real Numbers**

Using this chapter's own 1 g worked example above ($\approx 9\times10^{13}$ J), and converting using the standard equivalence of 1 kiloton of TNT ≈ 4.184×10^{12} J:

$$\text{Yield} = (9\times10^{13}) \;/\; (4.184\times10^{12})$$

$$\text{Yield} \approx 21.5 \text{ kilotons of TNT}$$

This matches the Trinity test's own real, reported yield of roughly 21 kilotons almost exactly — a genuine, checkable confirmation that a single missing gram of mass, converted via $E = mc^2$, really did account for the full explosive energy of the world's first nuclear detonation.

Three Real Scales, Compared

Example	Mass Converted	Energy Released
Uranium fission (general)	~0.1% of total mass	Real, usable nuclear power
The Sun (every second)	~4.26 billion kg	3.846×10^2&sup6; W
Trinity test (1945)	~1 gram	~21 kilotons of TNT

Hands-On Exercises

Exercise 1

Calculate the rest energy of 5 x 10^-4 kg (0.5 g) of matter, using E = mc^2.

→ Solution

Exercise 2

A photon has momentum p = 2 x 10^-27 kg.m/s and zero rest mass. Using E = pc, calculate its energy.

→ Solution

Exercise 3

Explain, in your own words, why Einstein's real 1905 paper deriving "mass decreases by L/c^2 when energy L is emitted" is a narrower claim than the modern, fully general E = mc^2 - and why this distinction matters even though the underlying physics is the same.

→ Solution

Quick Reference

- Einstein's real 1905 paper derived a mass CHANGE from emitted energy ($\Delta m = L/c^2$), not the compact absolute $E = mc^2$ formula itself
- **Full relation:** $E^2 = (pc)^2 + (mc^2)^2$ — reduces to $E = mc^2$ at rest, and $E = pc$ for massless particles
- Nuclear mass defect is the real, measured evidence: missing mass = binding energy, via $E = mc^2$
- The Sun's real 4.26 billion kg/s conversion rate matches $E = mc^2$ applied to its own measured power output almost exactly
- The Trinity test's real ~1 gram of converted mass matches its own reported ~21 kiloton yield almost exactly, via $E = mc^2$

Next chapter: the Capstone — General Relativity's real basic ideas, and gravity revisited a second time, closing this course's own full circle back to Classical Mechanics & Thermodynamics.

Electromagnetism & Relativity

Course 2 · Chapter 10 · Capstone: General Relativity's Real Basic Ideas, and Gravity Revisited a Second Time

This course has run one continuous real thread from Chapter 1's own point charges to Chapter 9's own atomic nucleus: electricity, unified with magnetism, unified with light, forced a rethink of space and time themselves. This closing chapter takes one further step Einstein himself took after 1905 — and closes the loop back to gravity, the very force this entire two-course project opened with.

The Course's Own Arc, in One Line

Charge (Ch.1) produces a field (Ch.2); current, a moving charge, produces a circuit (Ch.3) and a magnetic field (Ch.4); a changing magnetic field induces current in return (Ch.5); Maxwell unified all of it, and discovered light itself is the result (Ch.6); light's own fixed speed broke the ether (Ch.7); Einstein's relativity explained why (Ch.8); and mass itself turned out to be a form of energy (Ch.9). This capstone is where Einstein himself went next.

The Equivalence Principle

In 1907, working at the Swiss patent office in Bern, Einstein had what he later called, in his own real words, "the happiest thought of my life": "*If a man falls freely, he would not feel his weight. I was taken aback. This simple thought experiment made a deep impression on me.*" A person in true free fall feels no weight at all — genuinely indistinguishable, from the inside, from floating weightlessly in deep space with no gravity present.

Einstein generalised this real insight into the **equivalence principle**: a uniform gravitational field and a uniform acceleration are, for any observer sealed inside a windowless room, completely physically indistinguishable. By 1911, he had already used this single principle to predict two genuinely new effects — that clocks run at different rates in different gravitational strengths, and that light itself bends in a gravitational field — four years before finishing the full theory in 1915.

Spacetime Curvature: Gravity as Geometry

General relativity's own deepest real idea is that gravity is not a force pulling objects together at all, in the way Chapter 1's own Coulomb's law describes electric force. Instead, mass and energy genuinely curve the fabric of space and time itself, and objects simply follow the straightest possible path through that curved geometry — a path that looks, to an observer, exactly like being pulled by a force. Physicist John Wheeler's own later summary captures the real idea compactly: matter tells spacetime how to curve; spacetime tells matter how to move.

Real Confirmation: Eddington's 1919 Eclipse Expedition

General relativity predicted something genuinely testable: starlight passing close to the Sun should bend by 1.75 arcseconds — exactly twice the 0.83 arcsecond deflection Newtonian gravity alone would predict for light (if light, despite having no rest mass, were still treated as subject to ordinary gravitational attraction). The only way to test it was during a total solar eclipse, when stars near the Sun's own position become briefly visible.

On 29 May 1919, two real expeditions observed exactly that eclipse: Arthur Eddington and Edwin Cottingham from the West African island of Príncipe, and Andrew Crommelin and Charles Davidson (from the Greenwich Observatory) from Sobral, Brazil. Both real, independent measurements confirmed Einstein's own predicted deflection, not Newton's smaller value. When the results were formally announced in November 1919, Einstein became an international celebrity almost overnight — *The New York Times* covered it prominently, and a real, documented wave of popular science books followed, launching what historians now call the "Einstein boom."

Gravity Revisited a Second Time

Classical Mechanics & Thermodynamics Chapter 6, titled "Gravity Revisited," explained how Newton's own 1687 law of universal gravitation mathematically derived Kepler's three empirical laws — a deeper theory explaining an older, purely observational one. This chapter closes the same pattern one level deeper: general relativity does not discard Newton's real, still-useful $F = Gm_1m_2/r^2$ formula — it contains it. In the everyday case of comparatively weak gravity and comparatively low speeds (everything from a dropped apple to an orbiting planet), general relativity's own predictions reduce mathematically to Newton's exact same familiar law. Newton's gravity was never wrong; it was always the correct, practical approximation for the weak-field regime it was built to describe — exactly the same real relationship this course's own sibling course established between Newton and Kepler, one level further down.

> 🔗 **A Third Real Payoff for the Exact Same GPS Number**
>
> Chapter 8 of this course already introduced the real, working general-relativistic correction inside every GPS satellite: because a satellite sits in weaker gravity at altitude than a ground receiver, its clock genuinely gains roughly 45,850 nanoseconds per day relative to the ground — a real, measured, and continuously corrected-for consequence of exactly the spacetime curvature this chapter has just described. Weaker gravitational curvature means less severe time dilation, so a satellite's own clock runs measurably faster than a clock on Earth's own surface — the identical underlying physics behind Eddington's own bent starlight, now put to daily, practical use in every phone with a GPS chip.

Every Chapter, In One Arc

Chapter	What It Contributed
1 — Electric Charge & Coulomb's Law	Force between charges; the real, unpublished history behind it

2 — Electric Fields & Potential	Faraday's field concept; Volta's real battery
3 — Circuits	Current, Ohm's law, and its own real cold reception
4 — Magnetism	Ørsted's real, deliberate 1820 discovery
5 — Electromagnetic Induction	Faraday's disk; Henry's real delayed credit
6 — Maxwell's Equations	Light unified with electricity and magnetism
7 — The Real Crisis	The ether's real null result; the real Einstein correction
8 — Special Relativity	Time dilation and length contraction, and three real confirmations
9 — $E=mc^2$	Mass-energy equivalence, verified via the Sun and Trinity
10 — This Capstone	General relativity, and gravity's own second real revisit

Hands-On Exercises

Exercise 1

Calculate the ratio of the real general-relativistic light-bending prediction (1.75 arcseconds) to the Newtonian prediction (0.83 arcseconds). How close is this ratio to exactly 2, and why does this specific factor-of-two difference make the 1919 eclipse test such a clean way to distinguish between the two theories?

→ Solution

Exercise 2

Explain, in your own words, why a person in true free fall (like an astronaut in orbit) feels weightless according to the equivalence principle - and why this is genuinely different from simply "being far away from Earth's gravity."

→ Solution

Exercise 3

Trace the real three-step chain of increasingly deep gravity theories spanning both courses of this project: Kepler's purely empirical laws (Astronomy Fundamentals), Newton's explanatory law of universal gravitation (Classical Mechanics & Thermodynamics, Chapter 6), and general relativity (this chapter). For each step, state in one sentence what the newer theory explained that the older one could only describe.

→ Solution

Course Complete — And the Full Classical Physics Project

Electromagnetism & Relativity is now complete, 10/10 chapters — from Chapter 1's own point charges through this capstone's return to gravity, general relativity, and Einstein's own real 1919 global fame. This closes the full, two-course Classical Physics project begun with Classical Mechanics & Thermodynamics: 20 chapters in total, opening with SUVAT and Newton's three laws, and closing with spacetime curvature and a real, working correction running inside every GPS satellite on Earth.